LR parsing is the derivative of context free grammars

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The title says it all.

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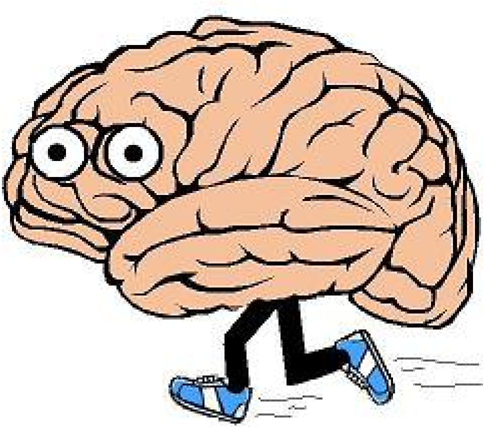
But you might not be convinced by me just saying so..

Jogging your brains



Context Free Grammars

A context free grammar

$$
\begin{array}{lcl}
E & \rightarrow & E * B \\
E & \rightarrow & E + B \\
E & \rightarrow & B \\
B & \rightarrow & 0 \\
B & \rightarrow & 1
\end{array}
$$

Formal Definition

A Context Free Grammar is a four-tuple:

- A set of terminals T
- A set of nonterminals NT
- A start symbol $S \in NT$
- A set of productions. Each production consist of:
    - A nonterminal; the head
    - The symbol $\rightarrow$
    - A body consisting of a sequence of terminals and nonterminals

Semantics of a grammar

The semantics of a grammar is defined in terms of derivation trees.

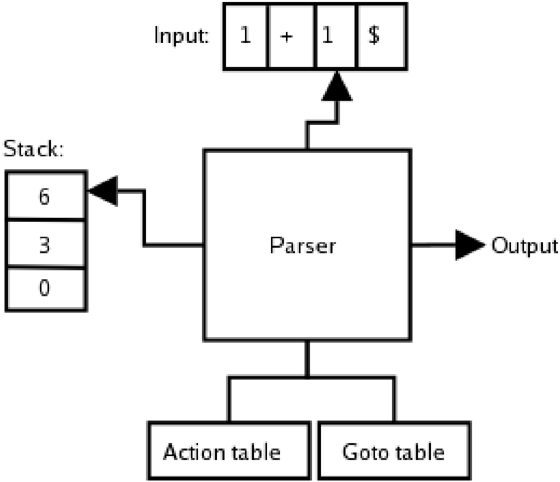
```
        E               E
        |               |
       /|\             /|\
      E * E           E + E
     /     \         /     \
    /|\     0       0      /|\
   0 + 1                  1 * 0
```

If a string has more than one derivation the grammar is ambiguous.

LR parsing

- The most common algorithm for parsing formal languages.
- Invented by Knuth
- Lots of tools for generating LR parsers: YACC, Bison, Happy, ...

LR parser machine



Producing the LR automaton

Going from a grammar to an LR automaton goes via item sets.

Item Set 0

$$
\begin{aligned}
E &\rightarrow \bullet E * B \\
E &\rightarrow \bullet E + B \\
E &\rightarrow \bullet B \\
E &\rightarrow \bullet 0 \\
E &\rightarrow \bullet 1
\end{aligned}
$$

Item Set 1

$$B \rightarrow 0\bullet$$

Item Set 2

$$B \rightarrow 1\bullet$$

Item Set 3

$$E \rightarrow E \bullet * B$$
$$E \rightarrow E \bullet + B$$

Advancing the dot one step is like recognizing one character in the input, alternatively a whole production.

Item set

Each item set corresponds to a state in the LR automaton (for LR(0) parsers).

Item sets are the derivative of the context free grammar

My claim:
Item sets are the derivative of the context free grammar

A new representation of Context Free Grammars

It's not exactly clear from the presentation of item sets that they correspond to some kind of derivative

To demonstrate this we need a new representation of context free grammars

A new syntax for syntax

$$
\begin{array}{lcl}
G, H \in \text{Grammar} & ::= & x \mid 0 \mid 1 \mid G \cdot H \mid G + H \mid T\ A \\
T \in \text{Grammar Tuples} & ::= & r \mid \mu r.\{A \mapsto G\}_{A \in N}
\end{array}
$$

An example grammar

$$\mu r.\{\quad \begin{array}{l} E \mapsto (rE\ \text{'+'}\ rB) + (rE\ \text{'*'}\ rB) + rB; \\ B \mapsto 0 + 1 \end{array} \qquad \}_{\{E,B\}}$$

$$\begin{array}{lcl} E & \rightarrow & E * B \\ E & \rightarrow & E + B \\ E & \rightarrow & B \\ B & \rightarrow & 0 \\ B & \rightarrow & 1 \end{array}$$

Semantics

- The current semantics I have is unfortunately not based on derivation trees.
- It only captures whether a grammar recognizes a language or not. There is no notion of ambiguity.
- $0, 1, +$ and $*$ form a semi-group

The Derivative

$$
\begin{array}{lcl}
\partial_x x & = & 1 \\
\partial_x (y \neq x) & = & 0 \\
\partial_x 0 & = & 0 \\
\partial_x 1 & = & 0 \\
\partial_x (G \cdot H) & = & \partial_x G \cdot H + \delta G \cdot \partial_x H \\
\partial_x (G + H) & = & \partial_x G + \partial_x H \\
\partial_x (rA) & = & 0 \\
\partial_x ((\mu r.\{A \mapsto G_A\}_{A \in N}) B) & = & \mu r.\{S \mapsto \partial_x G_B\}_{\{S\} \cup N} S
\end{array}
$$

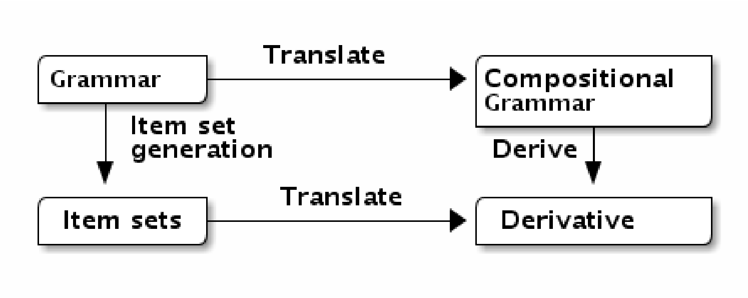
Auxiliary function, empty string recognition

$$\begin{array}{lcl}
\delta x & = & 0 \\
\delta 0 & = & 1 \\
\delta 1 & = & 0 \\
\delta(G \cdot H) & = & \delta G \cdot \delta H \\
\delta(G + H) & = & \delta G + \delta H \\
\delta(rA) & = & rA \\
\delta(\mu r.\{A \mapsto H_A\}_N A) & = & \mu r.\{A \mapsto \delta H_A\} A
\end{array}$$

Derivative wrt Non-terminals

$$
\begin{array}{lcl}
\partial_A x & = & 0 \\
\partial_A 0 & = & 0 \\
\partial_A 1 & = & 0 \\
\partial_A (G \cdot H) & = & \partial_A G \cdot H + \delta G \cdot \partial_A H \\
\partial_A (G + H) & = & \partial_A G + \partial_A H \\
\partial_A (rA) & = & 1 \\
\partial_A (rB \neq A) & = & 0 \\
\partial_A (\mu r.\{B \mapsto G_B\}_N B) & = & \mu r.\{B \mapsto \partial_A G_B\}_N B
\end{array}
$$

The Proof


Grammar
Translate
Compositional Grammar
Item set generation
Derive
Item sets
Translate
Derivative

Derivatives

The intuition behind derivatives in computer science: removing one item

- Data types: making one hole
- Regular expressions: recognizing one character
- LR parsing: recognizing one character

Questions

Questions?

The sequence rule

$$\partial_x(G \cdot H) = \partial_x G \cdot H + \delta G \cdot \partial_x H$$

$$\partial_x(G \cdot H) = \partial_x G \cdot H + G \cdot \partial_x H$$