ACL2 verification of Simplicial Complexes programs for the Kenzo system[1]

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- Kenzo
  - Symbolic Computation System devoted to Algebraic Topology

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  - Isabelle
  - Coq
  - ACL2

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  - ACL2 - simplicial structures

ACL2

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  - Programming Language
  - First-Order Logic
  - Theorem Prover

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- ACL2 (A Computational Logic for an Applicative Common Lisp)
- ACL2
  - Programming Language
  - First-Order Logic
  - Theorem Prover
- Proof techniques:
  - Simplification
  - Induction
  - *"The Method"*

Goal

- A new *certified* module for the Kenzo system
  - Simplicial Complexes

From "General" Topology to Homological Algebra

Topological Space

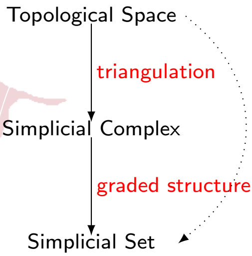
From "General" Topology to Homological Algebra

Topological Space

↓ triangulation

Simplicial Complex

From "General" Topology to Homological Algebra



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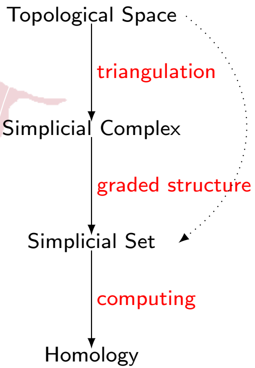

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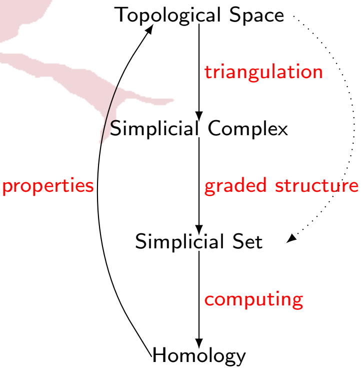
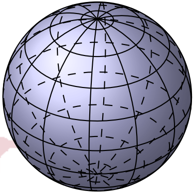


An example

Topological Space



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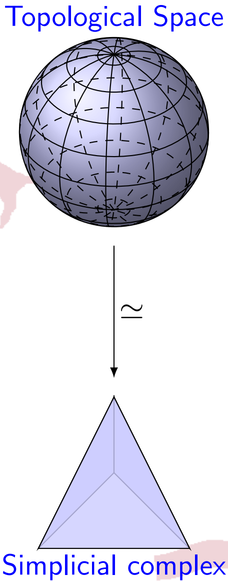

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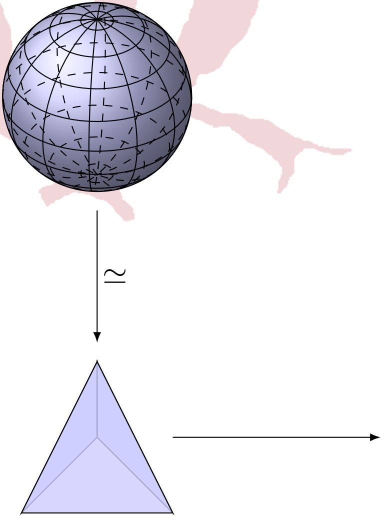


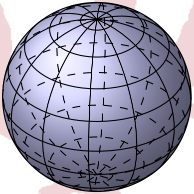
Simplicial complex

$$K_0 = \text{vertices (4 vertices)}$$
$$K_1 = \text{edges (6 edges)}$$
$$K_2 = \text{triangles (4 triangles)}$$

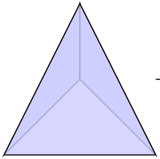
Simplicial set

An example

Topological Space



$\downarrow \simeq$



Simplicial complex

$\longrightarrow$

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Simplicial set

$\uparrow$

Homology groups

$$H_0 = \mathbb{Z}$$
$$H_1 = 0$$
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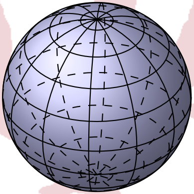
$$\ldots$$

An example

Topological Space



Homology groups

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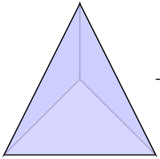

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- if σ_n is a simplex of C, and τ_p is a face of σ_n, then τ_p is in C;
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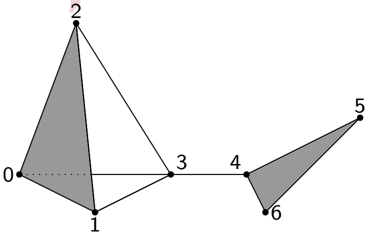
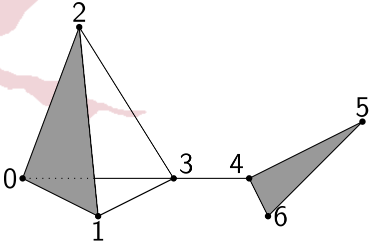


$C = \{\emptyset, \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\},$
$\{0,1\}, \{0,2\}, \{0,3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{3,4\}, \{4,5\}, \{4,6\}, \{5,6\},$
$\{0,1,2\}, \{4,5,6\}\}$

Simplicial Complexes

Facets: maximal elements of the simplicial complex



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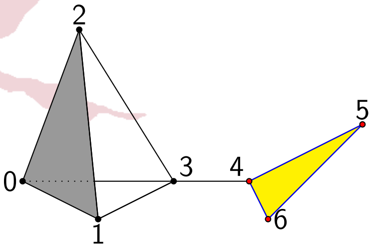

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$\{4,5,6\} \rightarrow \{\emptyset,\{4\},\{5\},\{6\},\{4,5\},\{5,6\},\{4,6\},\{4,5,6\}\}$

Simplicial Complexes

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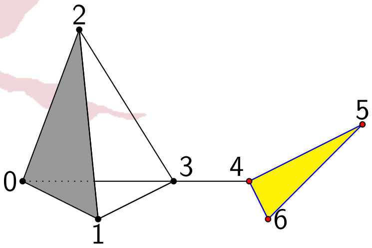

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Simplicial Complexes to Simplicial Sets

Simplicial Complex —graded structure→ Simplicial Set

Simplicial Complexes to Simplicial Sets

Definition

A *simplicial set* K, is a union $K = \bigcup_{q \geq 0} K^q$, where the K^q are disjoints sets, together with functions:

$$\begin{array}{lll} \partial_i^q : K^q \rightarrow K^{q-1}, & q > 0, & i = 0, \ldots, q, \\ \eta_i^q : K^q \rightarrow K^{q+1}, & q \geq 0, & i = 0, \ldots, q, \end{array}$$

subject to the relations:

$$\begin{array}{llllll} (1) & \partial_i^{q-1}\partial_j^q & = & \partial_{j-1}^{q-1}\partial_i^q & \text{if} & i < j, \\ (2) & \eta_i^{q+1}\eta_j^q & = & \eta_j^{q+1}\eta_{i-1}^q & \text{if} & i > j, \\ (3) & \partial_i^{q+1}\eta_j^q & = & \eta_{j-1}^{q-1}\partial_i^q & \text{if} & i < j, \\ (4) & \partial_i^{q+1}\eta_i^q & = & \textit{identity} & = & \partial_{i+1}^{q+1}\eta_i^q, \\ (5) & \partial_i^{q+1}\eta_j^q & = & \eta_j^{q-1}\partial_{i-1}^q & \text{if} & i > j+1, \end{array}$$

Simplicial Complexes to Simplicial Sets

Definition

Let C be a simplicial complex. Then the *simplicial set* $K(C)$ *canonically associated* with C is defined as follows. The set $K^n(C)$ of n-simplexes is the set made of the simplexes of cardinality $n+1$ of C. In addition, let a simplex $\{v_0, \ldots, v_q\}$ the *face* and *degeneracy* operators are defined as follows:

$$\begin{aligned} \partial_i(\{v_0, \ldots, v_i, \ldots, v_q\}) &= \{v_0, \ldots, v_{i-1}, v_{i+1}, \ldots, v_q\} \\ \eta_i(\{v_0, \ldots, v_i, \ldots, v_q\}) &= \{v_0, \ldots, v_i, v_i, \ldots, v_q\} \end{aligned}$$

Goals

- New Kenzo module:

Goals

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  - program1: facets $\longrightarrow$ simplicial complex
  - program2: simplicial complex $\longrightarrow$ simplicial set

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  - program2: simplicial complex $\longrightarrow$ simplicial set
- Certification of the correctness of the programs

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Program1: Simplicial Complex from facets

- `simplicial-complex-generator`:
  Input: a list of simplexes
  Output: a simplicial complex

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    Input: a list of simplexes
    Output: a list of simplexes with the properties of simplicial complexes but with duplicates

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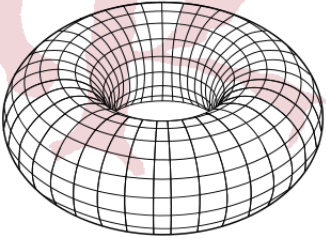
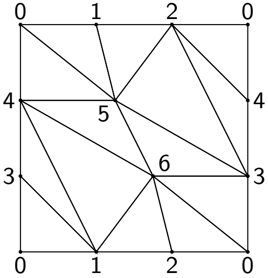
    Output: a list of simplexes with the properties of simplicial complexes but with duplicates
  - `simplicial-complex-generator-from-simplex`:
    Input: a simplex
    Output: a simplicial complex

Example

Torus simplicial complex $S^1 \times S^1$:






```
> (setf torus-sc (simplicial-complex-generator
'((0 1 3) (0 1 5) (0 2 4)
  (0 2 6) (0 3 6) (0 4 5)
  (1 2 5) (1 2 6) (1 3 4)
  (1 4 6) (2 3 4) (2 3 5)
  (3 5 6) (4 5 6))))
((0 1 3) (0 1) (0 1 5) (0 2 4) (0 2) ...)
```

Program2: Simplicial Set from Simplicial Complex

- `ss-from-sc`:
  Input: a simplicial complex
  Output: a simplicial set

Program2: Simplicial Set from Simplicial Complex

- `ss-from-sc`:
  Input: a simplicial complex
  Output: a simplicial set
  - Kenzo function `build-smst`:

```
(build-smst
  :basis basis
  :face face
  ...)
```

    - `basis`: a function returning the list of simplexes in a dimension
    - `face`: a function for face operation
    - `degeneracy`: not included

Example

Simplicial set canonically associated to `torus-sc`:

```
> (setf torus-ss (ss-from-sc torus-sc)) ✠
[K1 Simplicial-Set]
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Example

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```

```
> (basis torus-ss 1)
((0 1) (0 2) (0 6) (0 3) (0 5) (0 4) (1 5) (2 6) (1 2) (1 3) ...)
```

Example

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```
> (setf torus-ss (ss-from-sc torus-sc)) ✠
[K1 Simplicial-Set]
```

```
> (basis torus-ss 1) ✠
((0 1) (0 2) (0 6) (0 3) (0 5) (0 4) (1 5) (2 6) (1 2) (1 3) ...)
```

```
> (homology torus-ss 0 3) ✠
Homology in dimension 0:
Component Z
Homology in dimension 1:
Component Z
Component Z
Homology in dimension 2:
Component Z
```

$H_0(torus) = \mathbb{Z}$, $H_1(torus) = \mathbb{Z} \oplus \mathbb{Z}$ and $H_2(torus) = \mathbb{Z}$

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Problem with program1

`simplicial-complex-generator` program:

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Input of a list of 11613 simplexes:

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Error: Stack overflow (signal 1000)
[condition type: SYNCHRONOUS-OPERATING-SYSTEM-SIGNAL]
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`optimized-simplicial-complex-generator`:

Problem with program1

`simplicial-complex-generator` program:

- Follows simple inductive schemas
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Input of a list of 11613 simplexes:

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> (simplicial-complex-generator ...) ✠
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`optimized-simplicial-complex-generator`:

- Equivalent efficient program

Problem with program1

`simplicial-complex-generator` program:

- Follows simple inductive schemas
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Input of a list of 11613 simplexes:

```
> (simplicial-complex-generator ...) ✠
Error: Stack overflow (signal 1000)
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`optimized-simplicial-complex-generator`:

- Equivalent efficient program
- Memoization technique

Two equivalent algorithms for program1

Situation:

- `simplicial-complex-generator` program is
- `optimized-simplicial-complex-generator` program is

Two equivalent algorithms for program1

Situation:

- `simplicial-complex-generator` program is
  - specially designed to be proved;

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- `simplicial-complex-generator` program is
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  - programmed in ACL2 (and, of course, Common Lisp);

- `optimized-simplicial-complex-generator` program is
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- `simplicial-complex-generator` program is
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Situation:

- `simplicial-complex-generator` program is
  - specially designed to be proved;
  - programmed in ACL2 (and, of course, Common Lisp);
  - not efficient;
  - tested;
  - proved in ACL2.
- `optimized-simplicial-complex-generator` program is
  - designed to be efficient;
  - written in Common Lisp;
  - efficient;
  - tested;
  - unproved.

Two equivalent algorithms for program1

- `optimized-simplicial-complex-generator` "equivalent to" `simplicial-complex-generator`.

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Two equivalent algorithms for program1

- `optimized-simplicial-complex-generator` "equivalent to" `simplicial-complex-generator`.
- Not a proof of the equivalence
- Automated testing

```
(defun automated-testing ()
  (let ((cases (generate-test-cases 100000)))
    (dolist (case cases)
      (if (not (equal-as-sc (simplicial-complex-generator case)
                            (optimized-simplicial-complex-generator case)))
          (report-on-failure case)))
  )
```

A Common Lisp (but not ACL2) program

ACL2 definitions for Simplicial Complexes

Definition (Simplicial Complex)

A *simplicial complex* C is a finite set of simplexes satisfying the properties:

- if σ_n is a simplex of C, and τ_p is a face of σ_n, then τ_p is in C;
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- simplex: `simplex-p`

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- list of simplexes: `list-of-simplexes-p`
- without duplicates: `without-duplicates-p`

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- face: `subset-p`

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- face: `subset-p`
- member: `member-equal`

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- simplex: `simplex-p`
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- without duplicates: `without-duplicates-p`
- face: `subset-p`
- member: `member-equal`
- intersection: `intersect`

ACL2 theorems for Simplicial Complexes

Definition (Simplicial Complex)

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Lemma

Let ls be a list of simplexes, then (simplicial-complex-generator ls) builds a list of simplexes.

```
(defthm simplicial-complex-generator-theorem-1
  (implies (list-of-simplexes-p ls)
           (and (list-of-simplexes-p (simplicial-complex-generator ls))
                (without-duplicates-p (simplicial-complex-generator ls)))))
```

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Lemma

Let x be a simplex and ls be a list of simplexes, if x is in (simplicial-complex-generator ls) and y is a face of x, then y is in (simplicial-complex-generator ls).

```
(defthm simplicial-complex-generator-theorem-2
  (implies (and (simplex-p s1)
                (simplex-p s3)
                (list-of-simplexes-p ls)
                (member-equal s1 (simplicial-complex-generator ls))
                (subset-p s3 s1))
           (member-equal s3 (simplicial-complex-generator ls))))
```

ACL2 theorems for Simplicial Complexes

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- if σ_n and τ_p are simplexes of C, then $\sigma_n \cap \tau_p$ is a common face of σ_n and τ_p.

Lemma

Let x, y be simplexes and ls be a list of simplexes, if x and y are in ($\mathit{simplicial\text{-}complex\text{-}generator}$ ls), then $x \cap y$ is a common face of x and y.

```
(defthm simplicial-complex-generator-theorem-3
  (implies (and (list-of-simplexes-p ls)
                (member-equal s1 (simplicial-complex-generator ls))
                (member-equal s2 (simplicial-complex-generator ls)))
           (and (subset-p (intersect s1 s2) s1)
                (subset-p (intersect s1 s2) s2))))
```

ACL2 theorems for Simplicial Complexes

Theorem

Let ls be a list of simplexes, then (`simplicial-complex-generator ls`*) constructs a simplicial complex.*

Proof.

Apply the three previous lemmas □

Theorem for Simplicial Sets from Simplicial Complexes

Proving truthfulness of Kenzo statements like:

```
> (setf torus-ss (ss-from-sc torus-sc)) ✠
[K1 Simplicial-Set]
```

where `torus-sc` is a simplicial complex

Theorem for Simplicial Sets from Simplicial Complexes

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Theorem

Let sc be a simplicial complex, then (`ss-from-sc` sc) constructs a simplicial set.

Main Tools

Theorem

Let $\mathcal{K}$ be a Kenzo object implementing a simplicial set. If for every natural number $q \geq 2$ and for every geometric simplex gmsm in dimension q the following properties hold:

1. $\forall i, j \in \mathbb{N} : i < j \leq q \rightarrow \partial_i^{q-1} \circ (\partial_j^q gmsm) = \partial_{j-1}^{q-1} \circ (\partial_i^q gmsm)$,
2. $\forall i \in \mathbb{N}$, $i \leq q$: $\partial_i^q gmsm$ *is a simplex of* $\mathcal{K}$ *in dimension* $q - 1$,

then:

$$\mathcal{K} \text{ is a simplicial set.}$$

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- Generic instantiation tool:
  - Development of a generic theory
  - Instantiation of definitions and theorems for different implementations

F. J. Martín-Mateos, J. A. Alonso, M. J. Hidalgo, and J. L. Ruiz-Reina. A Generic Instantiation Tool and a Case Study: A Generic Multiset Theory. Proceedings of the Third ACL2 workshop. Grenoble, Francia, pp. 188–203, 2002.

Generic simplicial set theory

- Generic simplicial set theory for simplicial complexes:
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Let sc be a simplicial complex, then (`ss-from-sc` sc) constructs a simplicial set.

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ACL2 verification of Simplicial Complexes programs for the Kenzo system

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